Quantum Compactifications

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Based on “Linear Sigma Models with Torsion”
by Callum Quigley and S.S.

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Outline:

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- Knowns and desires

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- Non-compact examples

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- Non-compact examples
- Compact examples

Knowns and Desires

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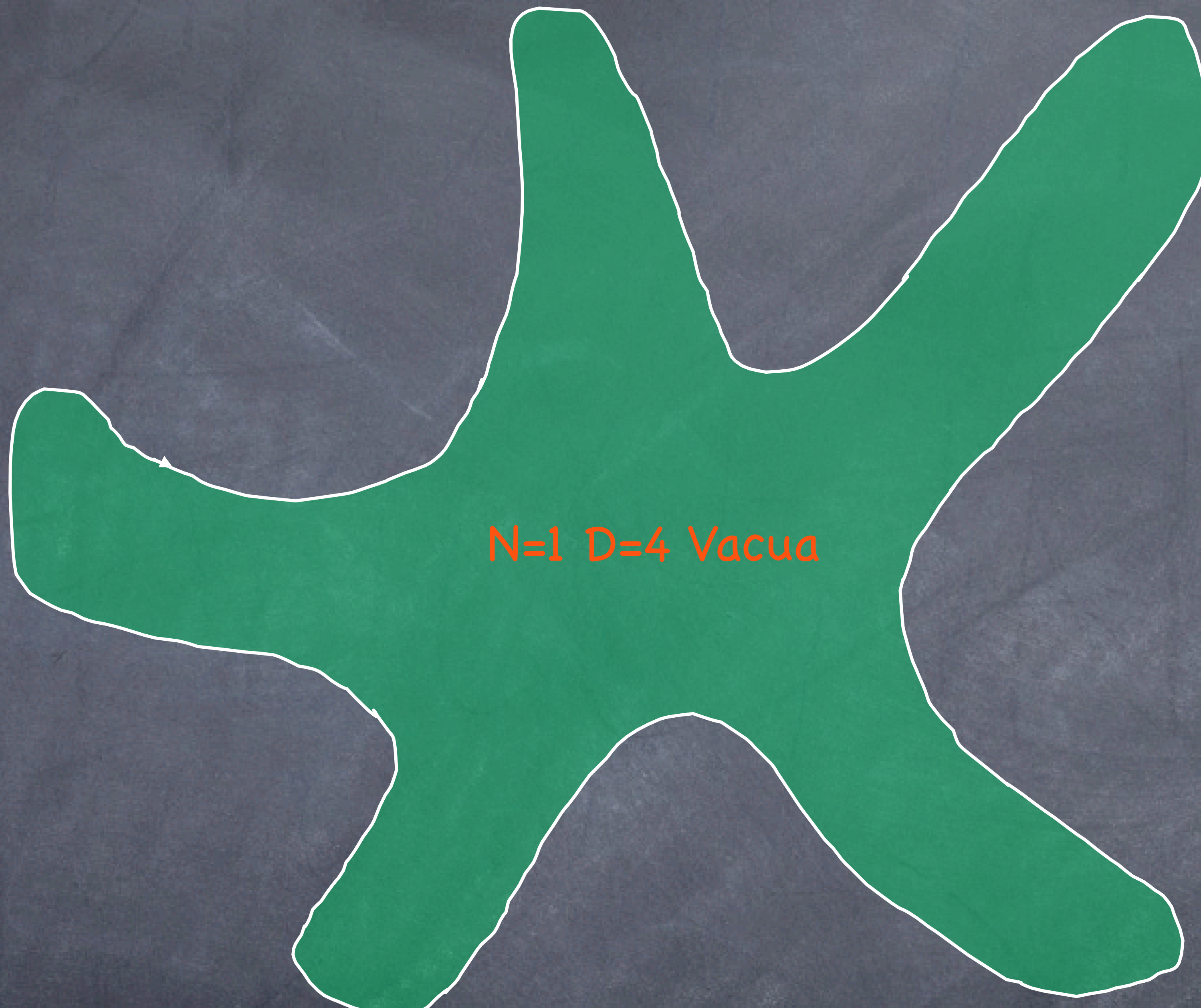
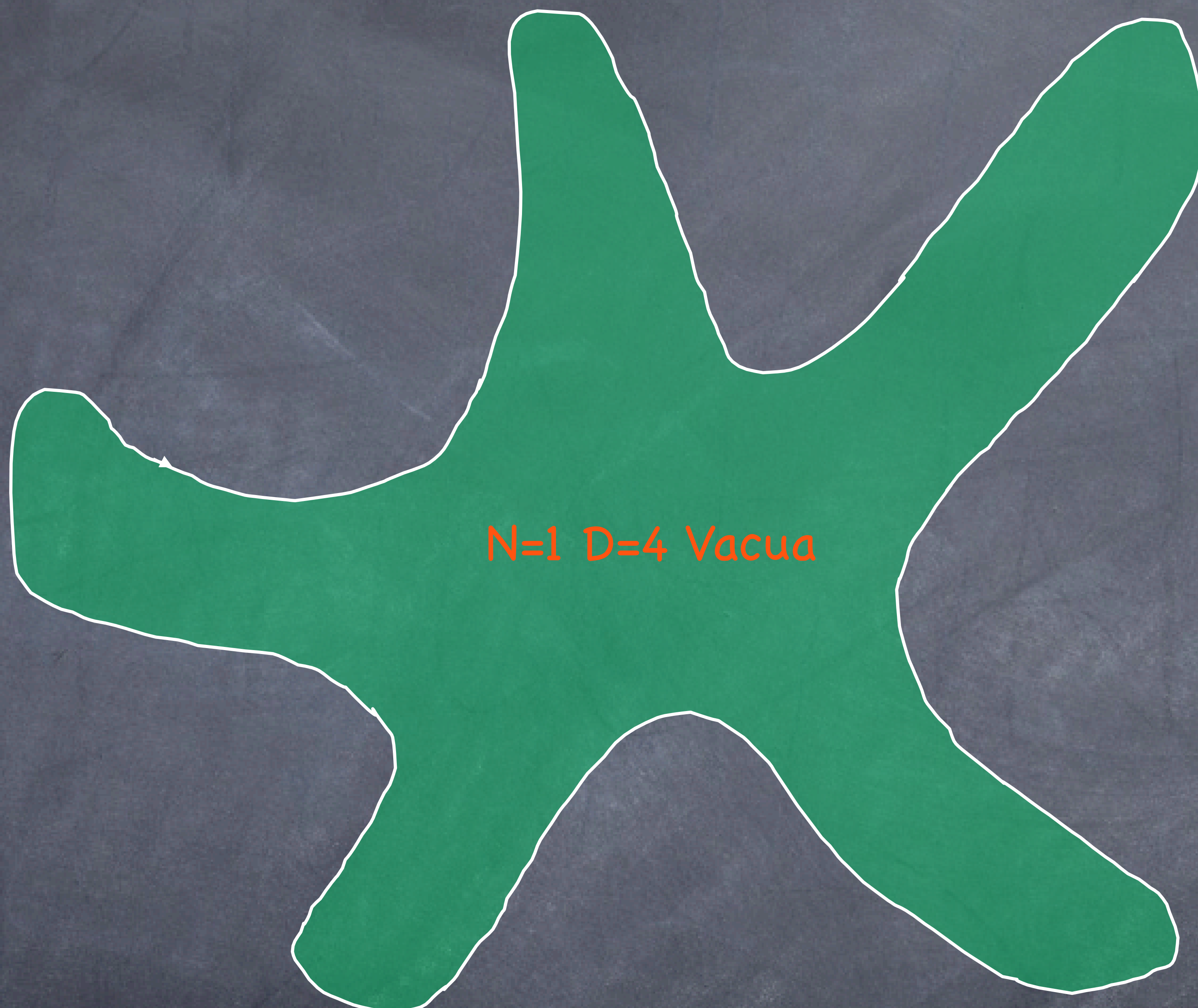
- The space of string compactifications is still largely mysterious.
- We need more powerful approaches to understand the interplay between cosmology, particle physics and Planck scale SUSY.


N=1 D=4 Vacua

Type I



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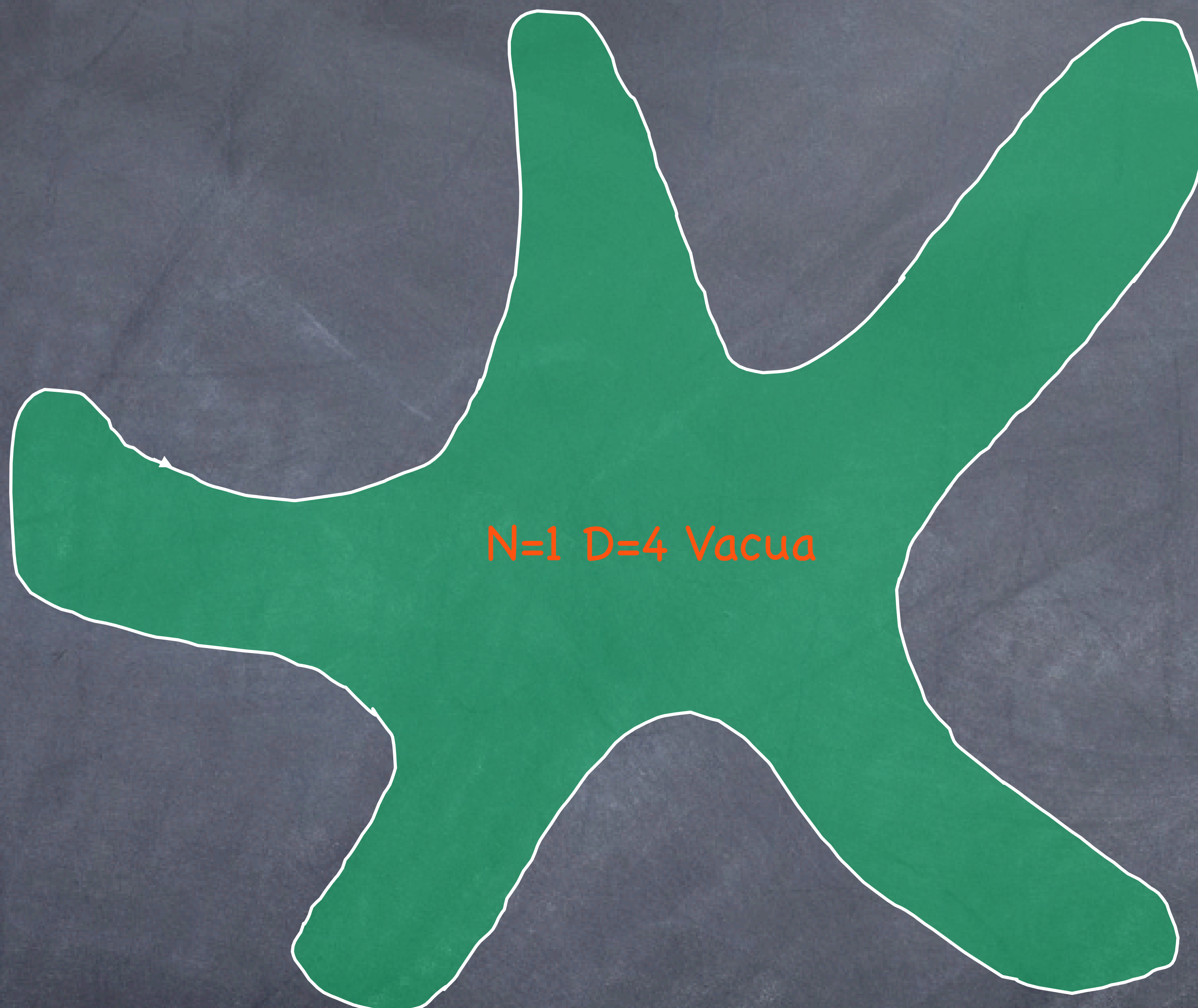
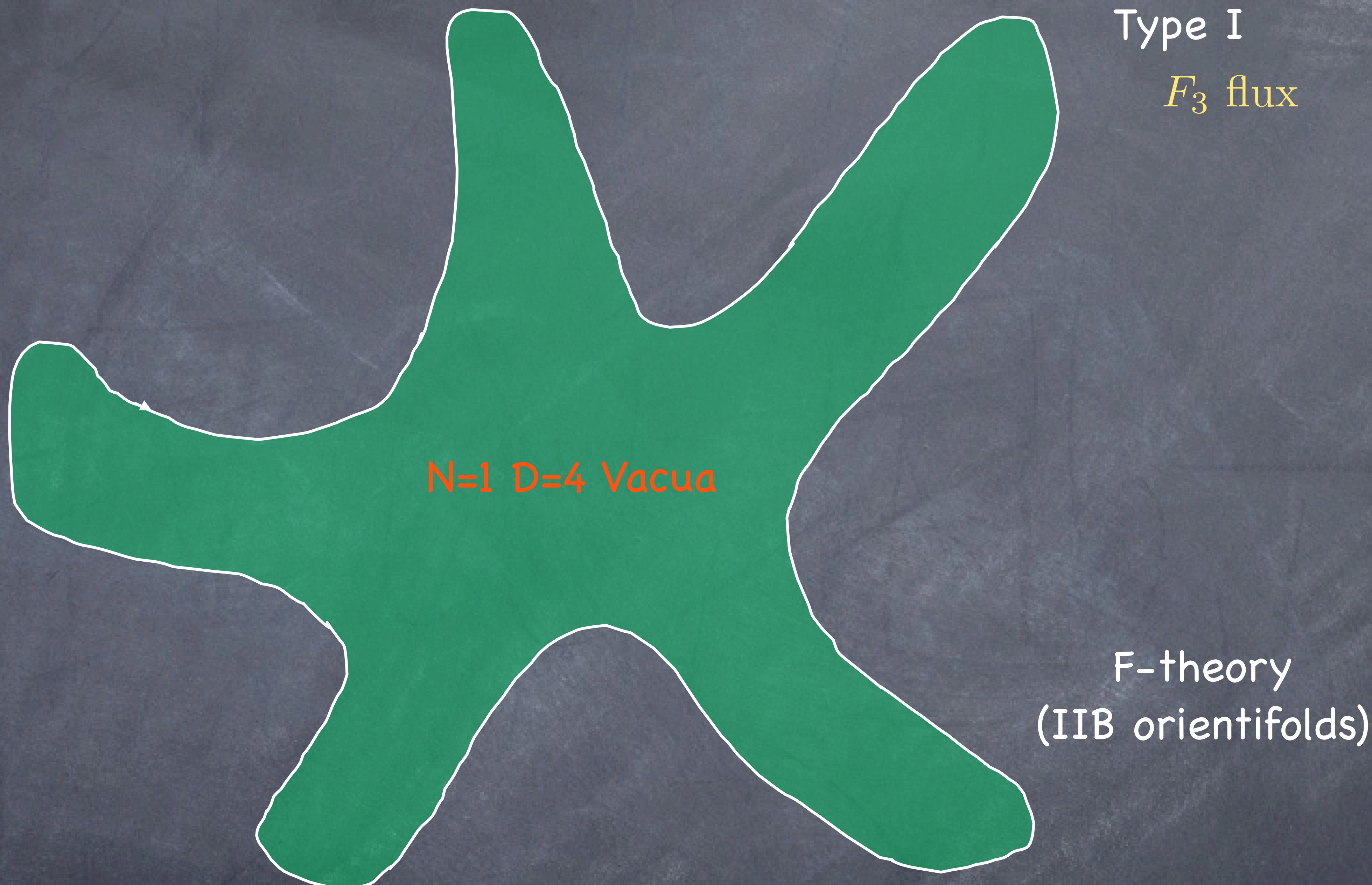
Type I

F_3 flux

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N=1 D=4 Vacua
F-theory
(IIB orientifolds)

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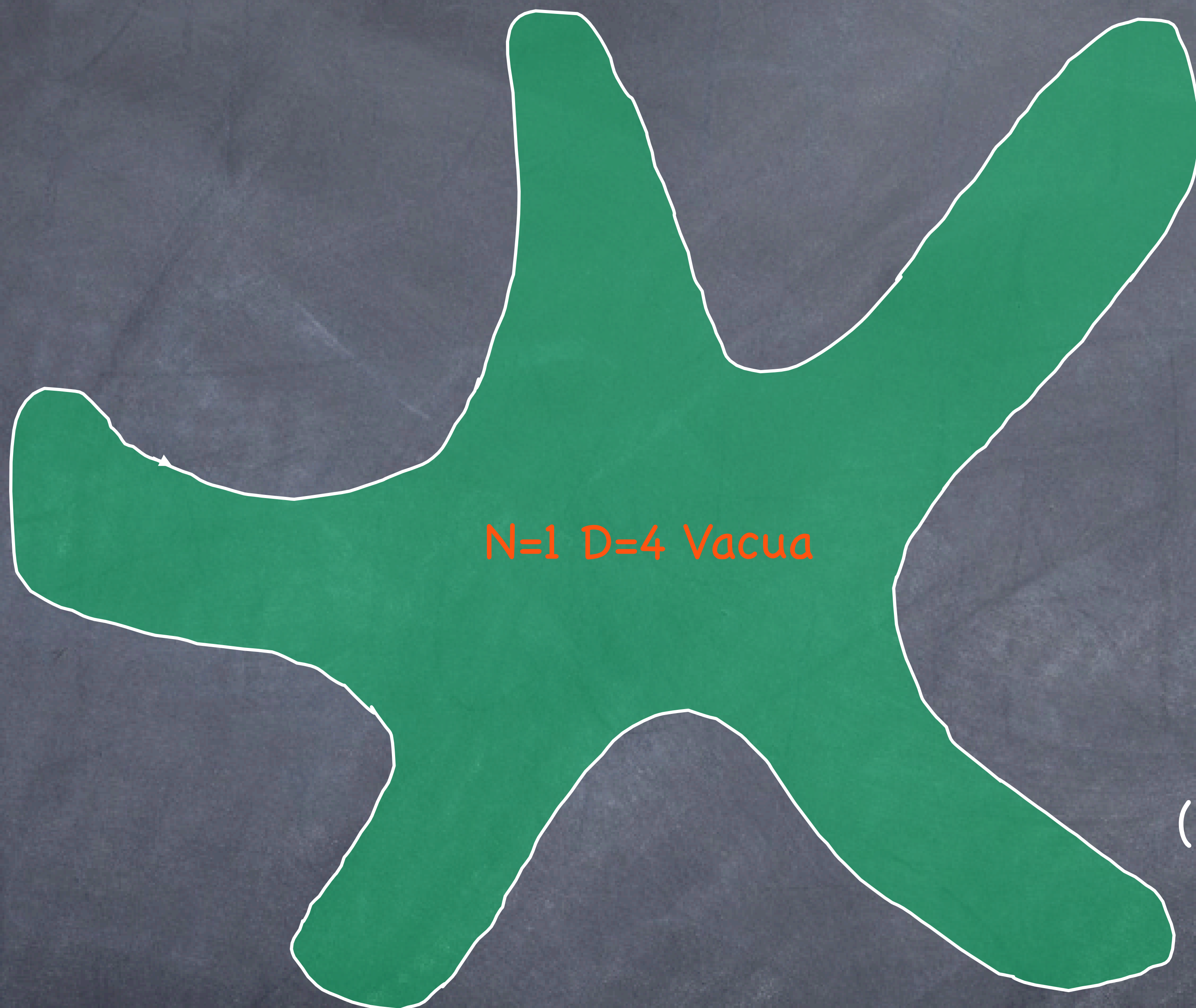

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G_3 flux

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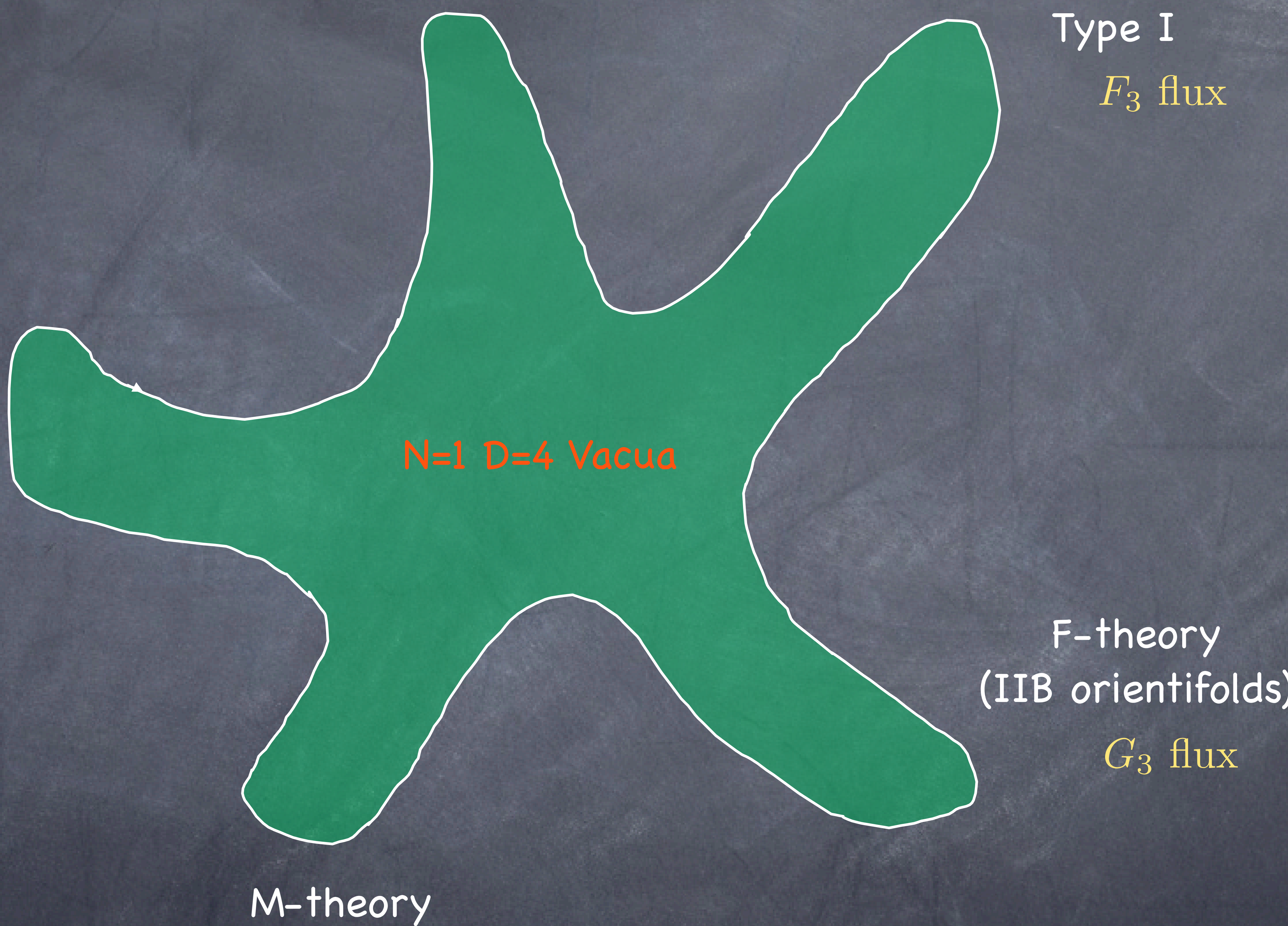
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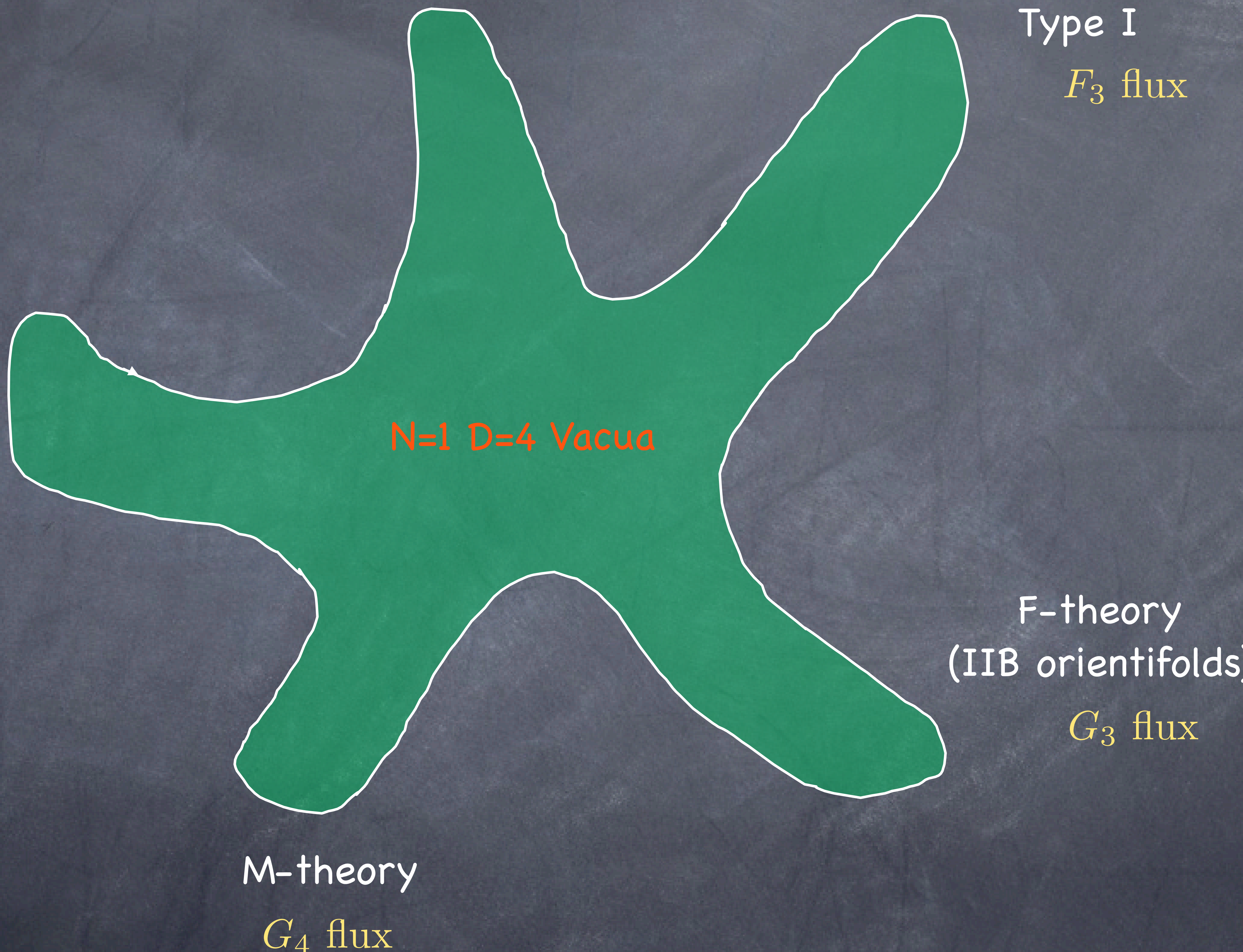
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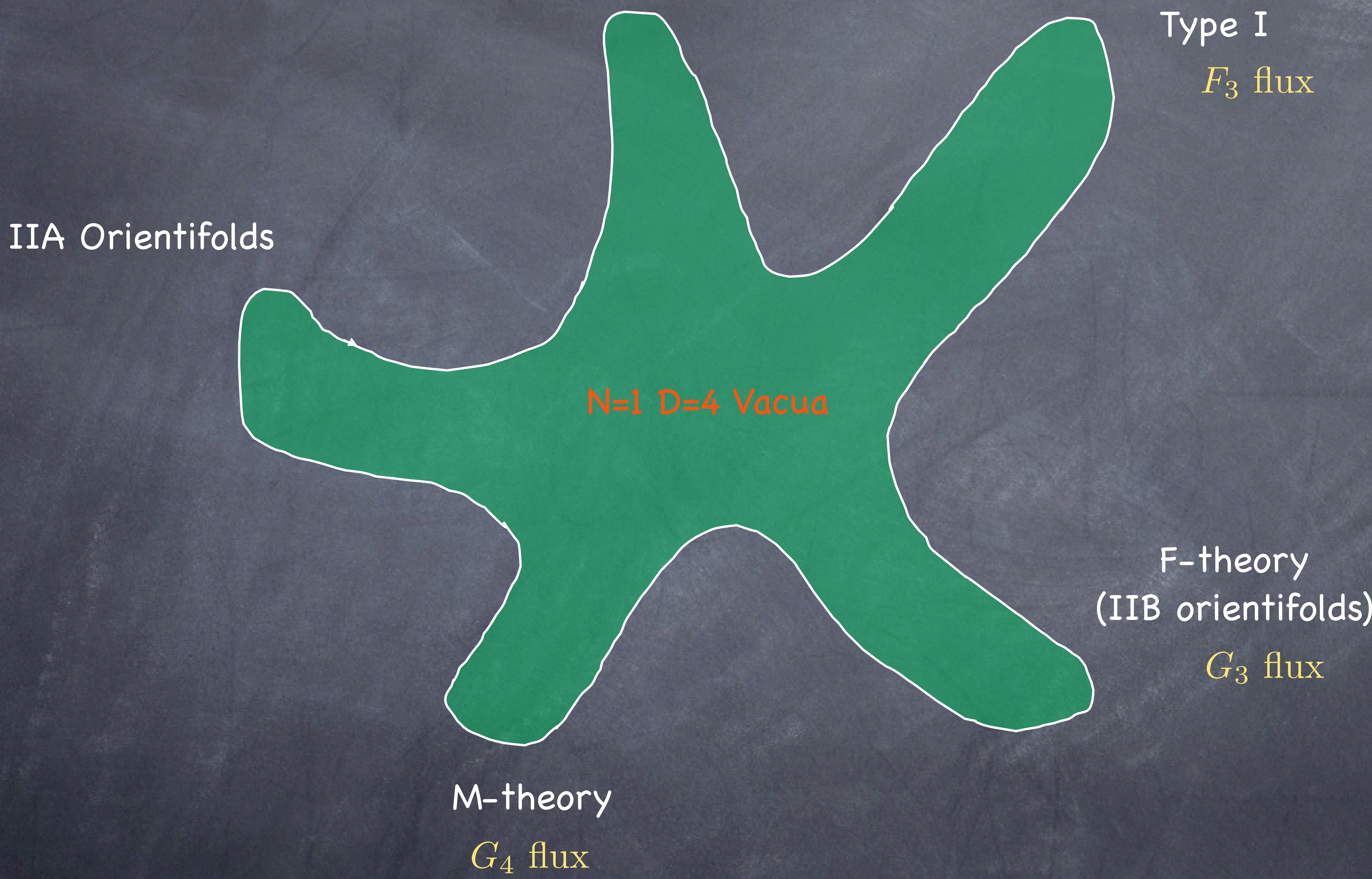
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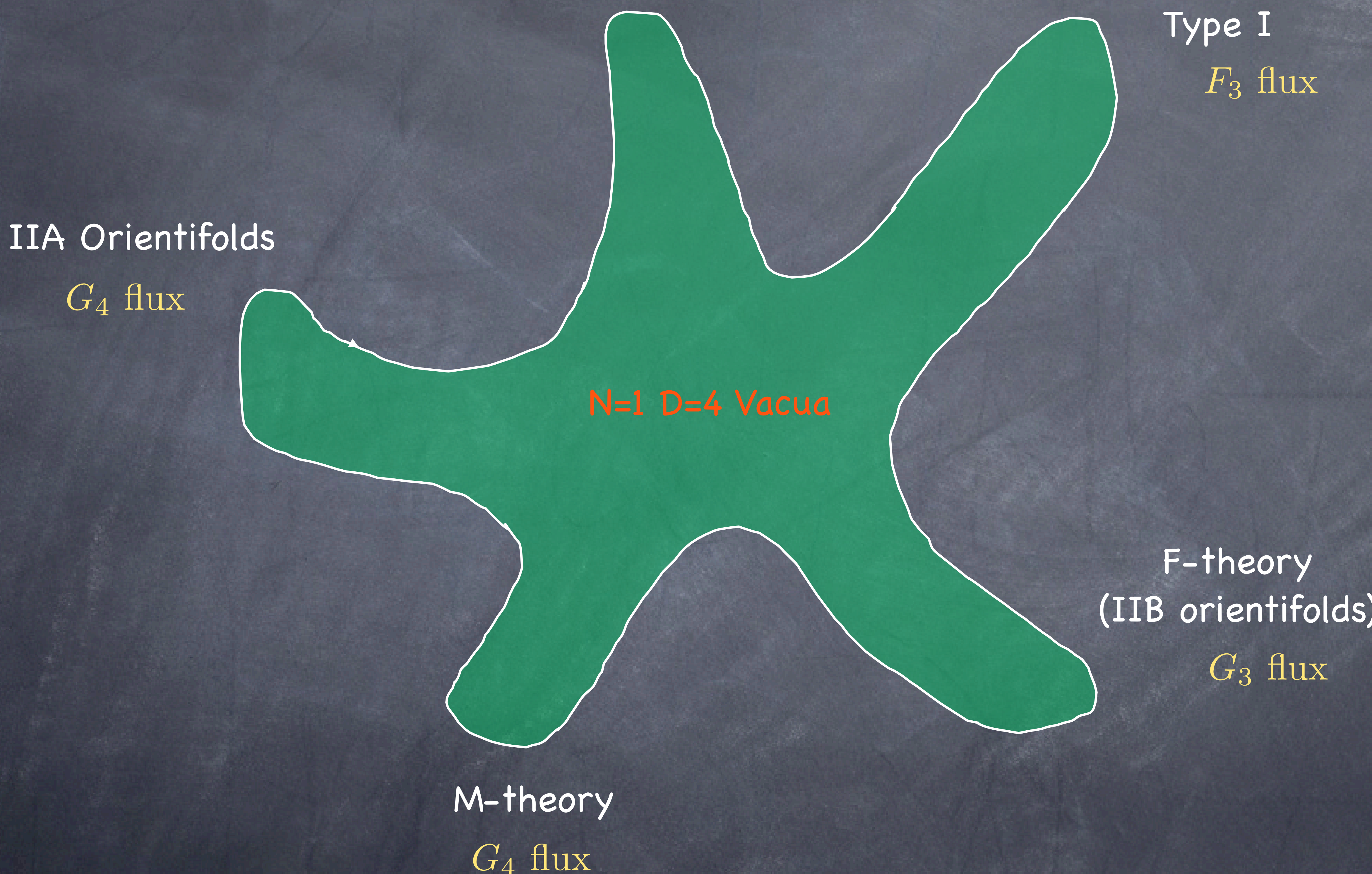

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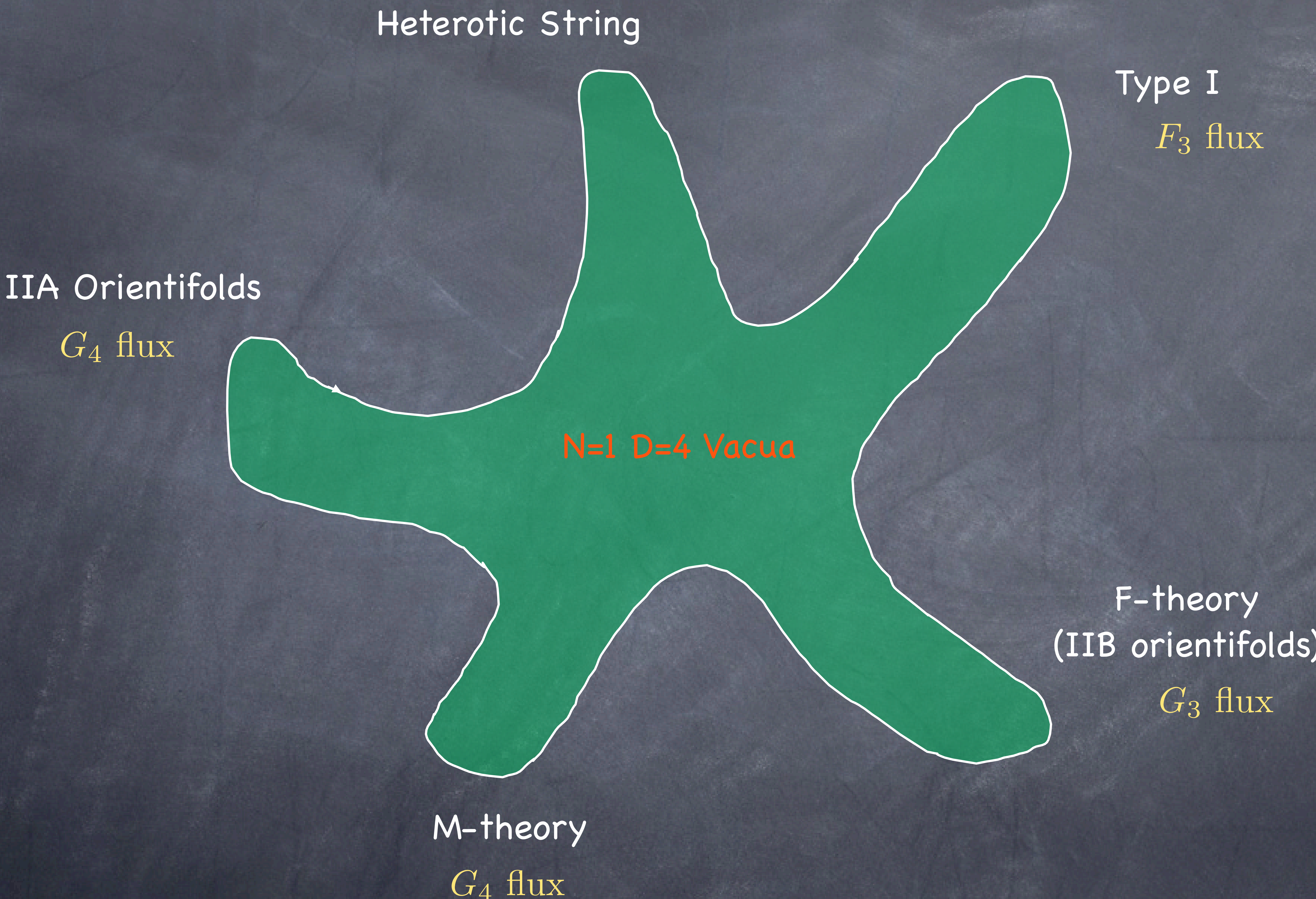
Heterotic String
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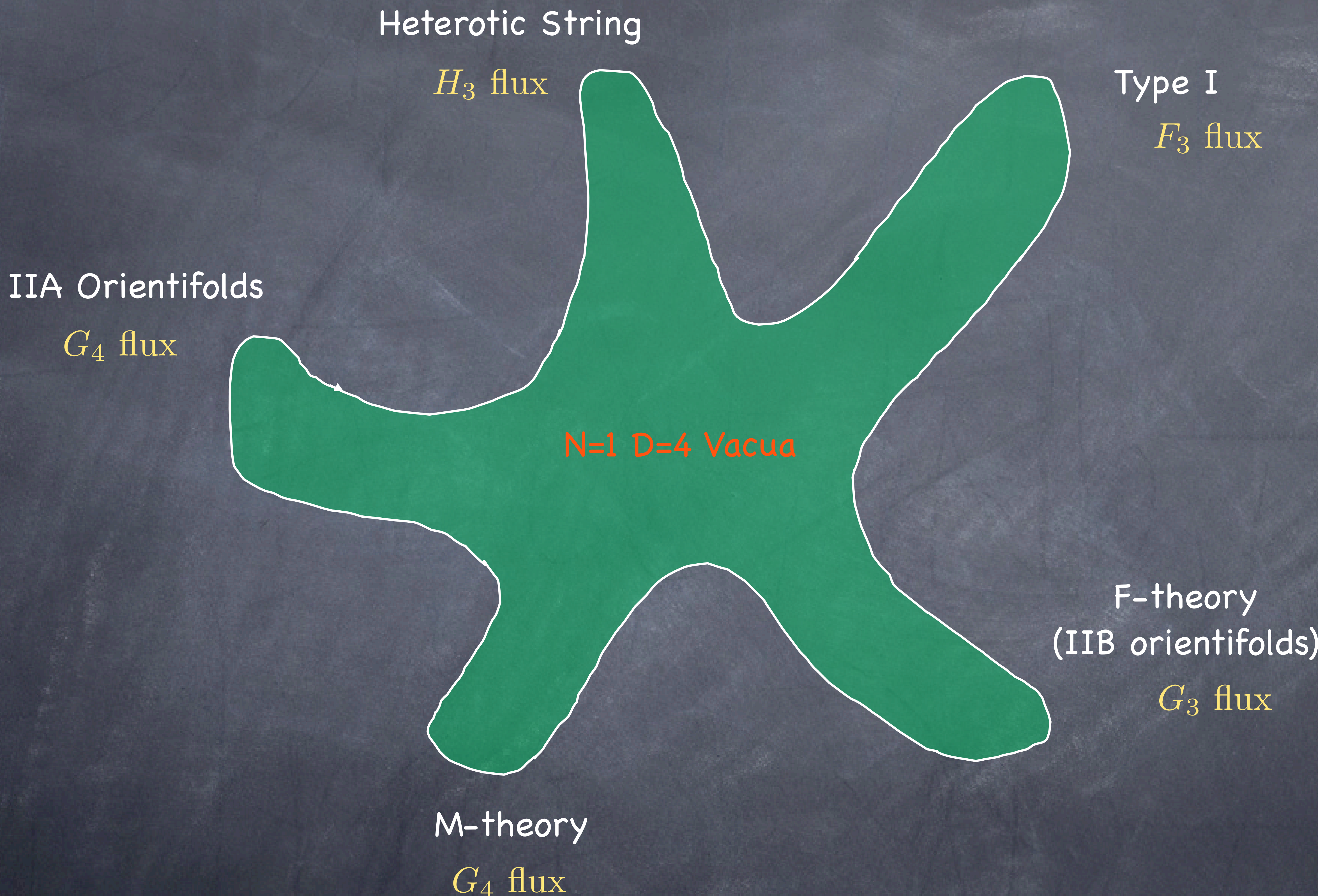
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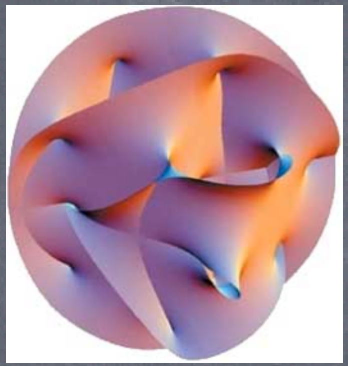

- Need to specify a metric and a choice of flux/gauge bundle.
- In every corner of the diagram, one finds the same qualitative physics: a landscape of SUSY vacua, potential large warping, etc.
- Only in the heterotic string is the required data purely NS with no RR fields. For models with RR fields, not much is known beyond the SUGRA approximation.

We will focus on N=1 SUSY heterotic string vacua.

Spacetime SUSY => (0,2) worldsheet SUSY.

In conventional models, this requires specifying a complex manifold,

$$J_{a\bar{b}} = i g_{a\bar{b}}$$

and a choice of H-flux and gauge-bundle:

$$H = i(\partial - \bar{\partial})J, \qquad g^{a\bar{b}} F_{a\bar{b}}$$

The primary constraint is the Bianchi identity which has a gravitational correction that plays the role of an orientifold source:

$$dH = \frac{\alpha'}{4} \left\{ \mathrm{tr}(R \wedge R)(\omega_+) - \mathrm{tr}(F \wedge F) \right\}$$

If H=0 at tree-level then the geometry is Ricci flat:

$$R_{\mu\nu} = 0$$

These spaces are Calabi-Yau and the most commonly studied compactifications.

They are likely to be a very special subset of generic heterotic compactifications which will typically have torsion:

$$R_{\mu\nu} \sim H_{\mu\rho\lambda} H_{\nu}^{\rho\lambda} + \dots$$

Generic compactifications should have few if any moduli other than the string dilaton.

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What we want: a linear framework analogous to Witten's linear sigma model that allows us to build analogues of the quintic Calabi-Yau.

$$\sum_i z_i^5 = 0 \subset \mathbb{P}^4$$

We will need to discover new geometries since very few examples of torsional spaces are known.

Non-Compact Models

Basics: we will restrict to (0,2) theories built from chiral superfields

$$\bar{D}_+\Phi^i = 0.$$

in a superspace with coordinates: $(\theta^+, \bar{\theta}^+)$.

Let's recall that the simplest (2,2) non-linear sigma models are defined by a choice of Kahler potential:

$$\mathcal{L} = \int d^4\theta\, K(\Phi, \bar{\Phi})$$

For a (0,2) theory, the analogous data is a collection of one-forms:

$$\begin{aligned}\mathcal{L} &\sim \int d^2\theta \, \left(K_i(\Phi,\bar{\Phi})\partial_- \Phi^i + c.c.\right) \\ &\sim -g_{i\bar{\jmath}}\,\partial_\alpha \phi^i \partial^\alpha \phi^{\bar{\jmath}} + b_{i\bar{\jmath}}\,\epsilon^{\alpha\beta}\partial_\alpha \phi^i \partial_\beta \phi^{\bar{\jmath}} + \ldots .\end{aligned}$$

$$g_{i\bar{\jmath}} = \partial_{(\bar{\jmath}} K_{i)}, \qquad b_{i\bar{\jmath}} = \partial_{[\bar{\jmath}} K_{i]}$$

The metric is generally non-Kahler.

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Linear models have canonical kinetic terms. Interactions are generated by gauging and introducing superpotentials.

To build a gauge theory, we introduce a chiral fermionic field strength

$$\Upsilon \sim \lambda + \theta^{+} \left(D - iF_{01} \right)$$

with couplings:

$$\mathcal{L}_{\Upsilon} \sim \frac{1}{e^2} \int d^2\theta \bar{\Upsilon} \Upsilon \sim \frac{1}{e^2} \left(\frac{1}{2} F_{01}^2 + i\bar{\lambda}\partial_{+}\lambda + \frac{1}{2} D^2 \right),$$

$$\mathcal{L}_{FI} \sim \frac{t}{4} \int d\theta^{+} \Upsilon + c.c. \sim -rD + \frac{\theta}{2\pi} F_{01}.$$

Taking $e \to \infty$, we can neglect the gauge kinetic terms

$$\mathcal{L}_{bosonic} = -|D_\mu \phi^i|^2 + \frac{\theta}{2\pi} F_{01} - V(\phi^i)$$

with a potential energy:

$$V = \frac{1}{2e^2} D^2, \qquad D = -e^2 \left(\sum q_i |\phi^i|^2 - r \right)$$

The moduli space is a toric variety:

$$D^{-1}(0)/U(1)$$

realized as a symplectic quotient of $\mathbb{C}^d$ by U(1) with moment map D.

In the IR limit, we can solve for the gauge field:

$$A_\mu = \frac{i}{2}\frac{\sum q_i\left(\bar\phi^i\partial_\mu\phi^i - \phi^i\partial_\mu\bar\phi^i\right)}{\sum q_i^2|\phi^i|^2}.$$

This gives the space-time B-field:

$$B = \frac{\theta}{2\pi}dA = \epsilon^{\mu\nu}B_{i\bar{j}}\partial_\mu\phi^i\partial_\nu\phi^{\bar{j}}$$

If we can make θ effectively vary, we can generate a non-zero H=dB.

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Modify the FI term which is a superpotential coupling:

$$\mathcal{L}_{FI} \sim \frac{t}{4} \int d\theta^+ f(\Phi) \Upsilon + c.c.$$

Gauge invariant

This has the following effect:

$$\frac{\theta}{2\pi} \to \frac{\theta}{2\pi} + \mathrm{Im}(f(\Phi))$$

$$V(\phi) \to \frac{e^2}{2} \left(\sum q_i |\phi^i|^2 + \mathrm{Re}(f) - r \right)^2$$

We generate a metric and H-field but these models are always non-compact.

Example: Conifold with Torsion

A single U(1) gauge group with charged matter:

$$\phi^i (i = 1,2) \quad q_i = +1, \qquad \phi^m (m = 1,2) \quad q_a = -1$$

$$|\phi^i|^2 - |\phi^m|^2 = r$$

Take a quadratic $f \sim f_{im}\phi^i\phi^m$. Higher powers are possible but the dilaton appears to blow up.

$$\phi_i = \bar{\phi}^i, \qquad \phi_{\bar{\imath}} = \phi^i, \qquad \tilde{\phi}_i = f_{im}\phi^m, \qquad \tilde{\phi}_{\bar{\imath}} = \bar{f}_{im}\bar{\phi}^m.$$

This leads to a B-field and metric which depend on a tunable deformation:

$$
\begin{aligned}
G_{i\bar{\jmath}} &= \delta_{i\bar{\jmath}} - \frac{\phi_i \phi_{\bar{\jmath}} - \widetilde{\phi}_i \widetilde{\phi}_{\bar{\jmath}}}{\sum |\phi|^2}, \\
B_{i\bar{\jmath}} &= -\frac{\phi_i \widetilde{\phi}_{\bar{\jmath}} - \phi_{\bar{\jmath}} \widetilde{\phi}_i}{\sum |\phi|^2}, \ldots
\end{aligned}
$$

This is a beautiful collection of non-compact torsional spaces.

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Compact Models

The previous approach never involves quantized fluxes yet we expect flux quantization to play a central role:

$$\frac{1}{2\pi\alpha'}\int H \in 2\pi\mathbb{Z}$$

How do we build compact models?

Let's draw an analogy with N=1 D=4 gauge theory:

$$\int d^2x d\theta^+ \, \Upsilon \quad \Leftrightarrow \quad \int d^4x \, d^2\theta \, W^\alpha W_\alpha$$

$$\mathrm{Im} \int d^4x \, d^2\theta \, (\tau W^\alpha W_\alpha) \quad \to \quad \tfrac{1}{4g^2} F^2 + \tfrac{\theta}{32\pi^2} F \wedge F$$

$$\tau = \tfrac{8\pi}{g^2} + i\theta$$

Renormalization is tightly controlled by holomorphy,

$$\Lambda^b \to e^{2\pi i} \Lambda^b, \qquad \tau \sim \tau + 1$$

$$\tau(\mu) = \tfrac{b}{2\pi i} \log(\Lambda/\mu) + f(\Lambda^b, \phi)$$

We will allow log interactions for Υ in the fundamental theory.

Note that no scale is need to define the log in two dimensions.

$$\mathcal{L}_{FI} = -\frac{i}{8\pi} \int d\theta^+ \, \underbrace{N_i^a}_{\text{Integers}} \log\left(\Phi^i\right) \underbrace{\Upsilon^a}_{\text{Different gauge factors}} + c.c.$$

We could also add additional single valued functions but let's focus on the log which has all the novelty.

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This model is not classically gauge-invariant! Under a $U(1)^b$ gauge transformation:

$$\Phi^i \to e^{iQ_i^b \Lambda^b} \Phi^i$$

$$\delta \mathcal{L}_{FI} = \left(\frac{N_i^a Q_i^b}{8\pi} \int d\theta^+ \, \Lambda^b \Upsilon^a + c.c. \right).$$

The antisymmetric part of this anomaly (in a,b) can be canceled by the classical coupling

$$\mathcal{L}_2 = \frac{1}{4\pi} \int d^2\theta^+ \, T^{ab} A^a V_-^b$$

where T^{ab} is antisymmetric. This coupling shifts by

$$\delta \mathcal{L}_2 = \left(-\frac{1}{8\pi} T^{ab} \int d\theta^+ \, \Lambda^a \Upsilon^b + c.c. \right).$$

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On the other hand, the gauge theory is generally anomalous with a symmetric one-loop anomaly:

$$\mathcal{A}^{ab} = \sum_i Q_i^a Q_i^b - \sum_\alpha Q_\alpha^a Q_\beta^b$$

Right-movers (curvature) Left-movers (NS5-branes & bundle)

$$\delta \mathcal{L} = \left(\frac{\mathcal{A}^{ab}}{8\pi} \int d\theta^+ \, \Lambda^a \Upsilon^b + c.c. \right).$$

Choosing

$$T^{ab} = Q_i^{[a} N_i^{b]}, \qquad \sum_i Q_i^{(a} N_i^{b)} + \mathcal{A}^{ab} = 0.$$

gives a quantum gauge invariant theory. These are intrinsically quantum models.

Let us see what is happening. Intuitively, the very large number of ways of canceling the gauge anomaly correspond to many geometries, fluxes and choices of gauge bundle that satisfy the Bianchi identity:

$$dH = \frac{\alpha'}{4}\left\{\mathrm{tr}(R \wedge R)(\omega_+) - \mathrm{tr}(F \wedge F)\right\}$$

This is a natural generalization of toric geometry and familiar toric spaces like projective spaces.

Let's get a feel for the structures that arise.

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The physical potentials are modified by the log interactions.

Take the case of one U(1) initially:

$$\sum_i Q_i|\phi^i|^2 + N_i \log|\phi^i| = r, \qquad Q_i > 0$$

If there are no log interactions, this would give a compact sphere. Quotienting by U(1) would then give weighted projective space.

If $N_i < 0$ then the space is still compact! The sign of the contribution to the gauge anomaly makes this appear "NS5-brane" like.

If some $N_i > 0$ then the space is non-compact and the anomaly contribution appears "anti-brane" like.

This situation is more involved with multiple U(1) factors.

Let's examine one case with U(1) X U(1):

$$Q_i^a = \begin{pmatrix} 1 & 1 & \dots & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 \end{pmatrix}$$

The first block has length n and the second length m with

$$m \geq n.$$

Add left-moving fermions with charges:

$$Q_m^\alpha = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 \end{pmatrix}.$$

$N_i^2 = -N_i^1 = 1$ for $i = 1, \dots, 2n$ and 0 otherwise.

$$\begin{aligned} \mathcal{A}^{ab} &= \sum_i Q_i^a Q_i^b - Q_m^\alpha Q_m^\beta = \begin{pmatrix} n & 0 \\ 0 & m \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & n+m \end{pmatrix} \\ &= \begin{pmatrix} n & 0 \\ 0 & -n \end{pmatrix} \end{aligned}$$

$$N_i^a Q_i^b = \begin{pmatrix} -n & -n \\ n & n \end{pmatrix}$$

So the anomaly can be canceled using AV couplings and not all N factors are negative. This is a mix of brane and anti-branes in the geometry.

Lastly, we can add superpotentials to carve out surfaces in these generalizations of toric varietes. Introduce a left-moving charged fermionic superfield:

$$\bar{\mathfrak{D}}_+\Gamma = \sqrt{2}E(\Phi).$$

The superpotential couplings

$$\mathcal{L}_J = -\frac{1}{\sqrt{2}}\int d\theta^+\,\Gamma\cdot J(\Phi) + c.c.$$

give a bosonic potential

$$V = |E|^2 + |J|^2.$$

For a suitable choice of fields and charges, these give conformal models generalizing Calabi-Yau spaces.

Summary

- There appear to be an enormous number of quantum consistent gauge theories.
- These theories provide a linear framework for constructing and studying flux vacua.
- The exploration has just begun!